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AUTHOR Sealey, Jean
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ABSTRACT

Research findings on problem solving and ideas for improving problem-solving instruction are presented. What a problem is and the fact that few problems appear in textbooks are noted. What successful problem solvers do is discussed, with the stages of problem solving described, including the point that checking their work may be the most important step for students. Four roles of the teacher in helping children at different levels are next discussed: modeling, helping or supporting, providing problems, and facilitating or sustaining interest. Features of a problem-solving program are then given, with comments on heuristics or strategies. An illustration of a classroom lesson is included in which the teacher serves as a model and a problem provider. (MNS)

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R & D INTERPRETATION SERVICE BULLETIN

MATHEMATICS

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Problem Solving

Jean Sealey

QUESTION: *How can teachers increase their students' ability and willingness to stick with solving problems, to think problems through, and to appreciate that alternative methods of solution do exist for most problems?*

Most definitions of problem solving have three common elements: an unrealized goal, an obstacle, and the use of reasoning to overcome the obstacle and reach the goal (6). The first two elements tell us that a person must be inherently interested in the solution, and the solution must not be immediately apparent.

Most problems that appear in textbooks are not problem-solving activities. Students have no inherent interest in the solution of textbook problems except to complete an assignment. Also, most textbook problems are clustered in such a way that the methods for their solution are obvious. For example, word problems that require using multiplication appear at the end of the unit on multiplication. If problems are arranged so that a variety of operations is required, key words often tell the student which operation to use. "How many are left?" clues the student to subtract. Textbook problems such as these are more accurately described as exercises.

How can you help your students increase their ability to solve problems? We can't give you a formula for teaching problem solving. We can share with you what researchers say about effective problem solvers. We can suggest ways you can help your students become more effective problem solvers.

OVERVIEW

Problem solving means different things to different people, but definitions of problem solving usually include a goal, an obstacle, and reasoning to overcome the obstacle. Skills to improve problem solving abilities can be taught, and by examining the characteristics of good problem solvers and their approach to the stages of problem solving, we can derive specific ways to help students learn these important skills. More information on problem solving is available in the *Research Within Reach* publications *Elementary School Mathematics* and *Secondary School Mathematics*, both published by the R & D Interpretation Service (1, 2). The numbers in parentheses correspond to the numbered references at the end of the article.

GOOD PROBLEM SOLVERS

PROBLEM SOLVING IS primarily a cognitive process. We can't observe it directly, but we can observe people as they solve problems. We can ask them to think aloud. Researchers have done just that. In general, successful problem solvers use a step-by-step approach. They are more active. They read, check, and review more; identify relevant information and use it; are able to plan an attack; use varied strategies; see the mathematical structure and can remember the structure of problems; and can see mathematical patterns and generalize across problems (1, 2). These are skills that can be learned. These are skills that teachers can help students develop.

STEPS IN PROBLEM SOLVING

WHEN WE BREAK PROBLEM SOLVING INTO smaller steps, we can look at strategies that are appropriate for each stage of the process. We can

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R & D Interpretation Service

AEL Appalachia Educational Laboratory
1031 Quarrier Street/P. O. Box 1348
Charleston, West Virginia 25325
304/347-0445

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U.S. Department of Education
Washington, D.C. 20202

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also identify the particular stage at which students are experiencing the most difficulty.

The first step in solving a problem is to recognize that there *is* a problem and to understand what the problem is. Understanding a problem has several facets. What information is relevant and what is irrelevant? What additional information is needed to solve the problem? Are there any unfamiliar words or phrases? What is the question? How will the answer be labeled? When students can answer these questions, they have an understanding of the problem.

The next step in solving a problem is to devise a plan. There are no algorithms for problem solving; rather, there are possible strategies we can try. The key to devising a plan is to be flexible and adaptive (5). You can teach students to consider several approaches: direct computation, trial and error, diagrams, equations, and charts or tables. All of these approaches are reasonable, but sometimes even a reasonable approach is inappropriate for a particular problem. Discussing strategies that don't solve the problem gives students valuable information, since knowing what *not* to do can be as helpful as knowing what to do.

When students understand the problem and have devised a plan, they execute the plan. They do it. If they decide that the problem can be solved by drawing a diagram, they draw the diagram. If they choose to do an experiment, they do the experiment. This is not the end of the problem, however. There's one more step.

"Checking their work may be the most important step for students in problem solving."

Looking back over what they've done and checking their work has been described by some as the most important step for students in problem solving (7). It may also be the most difficult one for you to teach since you can't predict students' solutions in advance. You can ask some general questions that will help students reflect on their work. "Why did you choose that particular strategy?" "Describe the procedure you used." "Does your solution answer the problem?" "Is this like any other problems you have worked?" "Are there any other strategies that would have worked?"

THE TEACHERS' ROLE

AS A TEACHER YOU CAN ASSUME FOUR roles in teaching problem solving (3). Each role is most appropriate for different levels of students' problem-solving ability.

- For students who have no experience with problem solving (Level 1), your role is to *model* successful problem solving. You can do this by thinking aloud. Demonstrate effective problem solving by telling students how you think through a problem. Read the problem aloud to your class. Identify the relevant information. Tell the class what information in the problem is needed to solve it and what information is extraneous. Reread the problem with only the relevant information. Consider several approaches or strategies. Tell why one may be more appropriate than others. Try the strategy you selected. Check your work to see if you executed your plan correctly. Ask yourself aloud if your solution answers the question. Talk through all these steps in detail. Our insert contains an example of how to model successful problem solving.

"As a teacher you can assume four roles in teaching problem solving."

You can also give students a chance to model effective problem solving for each other. Assign a problem to two students or to a small group of students. Allow them to work out the solution to the problem together, sharing their ideas and strategies. Thinking aloud gives students a chance to see the gaps in their thinking. Hearing the approaches of others makes them aware of the variety of possible solutions. It gives them an opportunity to check their own approaches to problems.

- As students progress in their problem-solving ability, they understand the strategies involved, but they lack confidence in their ability to solve problems independently (Level 2). Your role is that of a *prosthesis*, or helper; you can give support and encouragement as students work together in pairs or small groups. You can also give students a checklist of things to try. Researchers call these suggestions "heuristics." Heuristics can be provided for each stage of the problem-solving process. This checklist of possible approaches gives the student a place to start. For example, diagrams are helpful to many people. In fact, when gifted 6th grade students in a recent study were asked what they thought made word problems difficult for average ability students, the lack of a diagram was the most frequent response. The researchers tested and confirmed the students' hypothesis. Most students, however, don't know how to draw an appropriate diagram so teaching diagraming can improve students' problem-solving skills. Diagrams help because they restate the problem in an uncluttered form and allow students to consider the problem outside the context of words. Diagrams also allow students to estimate a solution or to find a solution not initially suggested by the problem (2).
- Eventually, students gain confidence and begin to offer novel solutions to problems (Level 3). At this point, your role is that of a *problem provider*. You can give students opportunities to practice problem solving with a variety of problems. Can you imagine learning

to play golf or to ski by listening to lectures without the opportunity to practice? You've demonstrated successful problem solving. You've given your students strategies to try. Now give them problems to solve.

• Finally, students are able to choose strategies that work, and they are successful in working most of the problems presented (Level 4). These students not only look for alternate solutions to problems that have been worked, they also look for new problems. Now your role is that of a *facilitator*, helping sustain students' interest in problem-solving activities. In this role, you will gradually reduce your involvement. Problem solving is an independent activity. When students are introduced to problem solving, they are dependent upon you for demonstrations, for strategies to try, and for providing problems. Your goal is to help the student internalize the strategies. You become less and less visible. For example, when a student makes a mistake, don't correct it or give him the appropriate strategy right away. Ask the student probing questions that are directed toward a more appropriate solution: "Why did you do it this way?" "Is this like another problem you've worked?" "What worked in another problem that might work here?" "What would happen if you tried another approach?"

FEATURES OF A PROBLEM SOLVING PROGRAM

RECENT INTEREST IN PROBLEM SOLVING has been accompanied by more suggestions for teaching problem solving. Some of these suggestions will be effective — some will not. As you examine materials and programs, there are some features that are supported by research (6). You will want to include these features in your problem-solving program.

An effective problem-solving instructional program will include problems that deal with situations familiar to the students. For very young students, solutions may involve counting their fingers, buttons, marbles, and other tangible objects. For older students, problems that deal with finding a bargain in clothes or car purchases are relevant. Problems related to sports, game strategies, and current events are realistic for older students as well.

Effective problem-solving programs also apply strategies to a variety of situations. Students are taught to apply the techniques they have learned to more than buttons and marbles. A program that extracts problems from the students' reality will give students a variety of situations in which to practice their new skills.

An opportunity to practice is the third feature of an effective problem-solving program. Learning any new skill, whether it's riding a bicycle or solving problems, requires practice.

Heuristics

Algorithms are specific, fixed steps that, when applied appropriately and executed correctly, assure the successful solution of a problem. Addition, subtraction, multiplication, and division are examples of algorithms. There are no algorithms for problem-solving. Instead, we can teach students heuristic. Heuristics are guidelines or suggestions that may be helpful in solving problems. Thinking of problem solving as a four-stage process is itself a heuristic (5).

To teach students to *obtain a better understanding of the problem*, tell them to try the following strategies: Rephrase the problem in your own words. Identify the relevant information. Determine if additional information is needed. Estimate a reasonable answer.

To teach students to *devise an appropriate plan of attack*, tell them to try the following heuristics: Look for similarities between this problem and others you have solved. Draw a picture or a diagram. Break the problem into parts. Simplify the problem, if possible, and solve the simpler problem. Organize the information. Look for a pattern. Try to write a mathematical sentence (for example, a + b = ____).

To teach students to *carry out the plan* efficiently, show them how to assure proper execution. Record what you have done. Solve the mathematical sentence if you wrote one. Complete the diagram. Fill in the blanks. Do each step.

To teach students to *check over their work*, suggest that they try the following steps. If possible, check your answer. Make sure your answer is in the form appropriate to the question (are the units those required in the problem?). Ask yourself if you used all the information given in the problem. If you didn't, why not? Ask yourself if this is the only possible answer to the question. Use your solution to this problem to solve a similar problem.

Finally, you will want a problem-solving program that requires students to communicate what they have learned with others. This means two-way communication between you and the students. Exclusive reliance on worksheets and lectures denies students the opportunity to talk about mathematics and to hear the ideas of others. Understanding mathematics, and problem solving in particular, is made easier when a teacher makes it an active, interactive process by "using materials, games, and the learners' natural curiosity" (6).

SUMMARY

YOU CAN HELP YOUR STUDENTS LEARN critical problem-solving skills by adapting your role to their needs. You begin by modeling effective problem-solving skills. Then you teach students strategies they can use. You provide problems for

them to solve, and you provide encouragement and support to maintain their interest. Whether you design your own problem-solving program or choose a commerically-available program, you will want to include problems that relate to students' reality in a variety of situations. Your students will have many opportunities to practice their skills and to communicate with you and with each other about what they have learned.

Problem-solving skills equip students to work problems in mathematics classes. The same skills — analyzing the situation, organizing information, planning an attack, executing the plan, and checking steps — equip students to deal with life outside the mathematics classroom.

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PROBLEM SOLVING

Teachers may assume four roles as they teach problem solving. These roles and the students' level of development for which each is most appropriate are described in the accompanying bulletin. Here we will expand on two of these roles: model and problem provider.

The Teacher as a Model

Let's look at a problem you might give third or fourth grade students.

Everyone in Mrs. Truman's 4th grade class is trying to guess how old she is. Mrs. Truman gave the class these clues:
Her age is between 1 and 50.
Her age is an odd number.
She graduated from college when she was 20.
When Mrs. Truman is 40, her daughter will be 17.
The sum of the digits in Mrs. Truman's age is 7.
Can you help the class determine Mrs. Truman's age?

Now let's see how you might think through this problem with your class. First, read the problem slowly and carefully aloud. Then talk through it step by step.

STEP 1: UNDERSTANDING THE PROBLEM

"The first thing I need to do is make sure I understand the problem. What is it I want to know? I want to know Mrs. Truman's age. The answer to this problem will be a number of years."

"What information does the problem give me?"

[*Write the information on the chalkboard.*]

1. Mrs. Truman teaches 4th grade.
2. Mrs. Truman's students want to know how old she she is.
3. Mrs. Truman gave the class clues to her age.
4. Mrs. Truman is older than 1 and younger than 50.
5. Mrs. Truman's age is an odd number.
6. Mrs. Truman graduated from college when she was 20.
7. When Mrs. Truman is 40, her daughter will be 17.
8. The sum of the digits in Mrs. Truman's age equals 7 (_____ + _____ = 7). I can write a mathematical sentence.

"What information do I need to find Mrs. Truman's age?

[*Read each piece of information and make a decision.*]

1. This is interesting, but it won't help me solve the problem.
2. This information will not help me solve the problem.
3. Well, I do need clues. Let's look at them.
4. Yes, I need to know this. This sets the limits within which Mrs. Truman's age lies. She is between 1 and 50. I know that she is not 1 or 50 because her age is *between* these two values.
5. Yes, I need this. I can eliminate all even numbers between 1 and 50.
6. This tells me that Mrs. Truman is older than 20 because that was how old she was when she graduated from college.
7. I need this. It tells me that Mrs. Truman is not yet 40. If I had information about how old Mrs. Truman's daughter is now, or how old Mrs. Truman was when her daughter was born, then her daughter's age, or the difference between their ages, would be very helpful. For now, though, I will set this information aside.
8. The sum of the digits in Mrs. Truman's age equals 7.

"Do I need additional information? Well, I may, but it looks as though I have enough information to devise a plan."

STEP 2: DEVISING MY PLAN

"I know that the number is between 1 and 50. I can make a list of all the numbers between 1 and 50 and then consider each clue. I will reject numbers that do not meet the conditions."

STEP 3: CARRYING OUT MY PLAN

"I will list the numbers 2, 3, 4 . . . 49.

[*Do this on the chalkboard.*]

"I can cross out all the even numbers because Mrs. Truman's age is an odd number.

[*Cross out all the even numbers on the chart.*]

"I can cross out 20 and all numbers less than 20 because I know Mrs. Truman has graduated from college, and she was 20 then.

"I can cross out 40 and all numbers more than 40 because I know Mrs. Truman is not yet 40."

~~2~~ ~~3~~ ~~4~~ ~~5~~ ~~6~~ ~~7~~ ~~8~~ ~~9~~ ~~10~~ ~~11~~ ~~12~~ ~~13~~
~~14~~ ~~15~~ ~~16~~ ~~17~~ ~~18~~ ~~19~~ ~~20~~ (21) ~~22~~ 23 ~~24~~ 25
~~26~~ 27 ~~28~~ 29 ~~30~~ 31 ~~32~~ 33 ~~34~~ 35 ~~36~~ 37
~~38~~ (39) ~~40~~ ~~41~~ ~~42~~ ~~43~~ ~~44~~ ~~45~~ ~~46~~ ~~47~~ ~~48~~ ~~49~~

"Now I have all odd numbers between 21 and 39.

"My final clue tells me that the sum of the digits is 7. I'll start with 21. 2 + 1 = 3. Now 23. 2 + 3 = 5. Next, 25. 2 + 5 = 7. Mrs. Truman must be 25!"

STEP 4: CHECKING MY STEPS

"But let me make sure my answer is correct. First, I know that my solution is in the correct units — years. The question asked for Mrs. Truman's age.

"Does my answer meet all the conditions of the clues Mrs. Truman gave her class?

- Is my answer between 1 and 50? Yes, 25 is between 1 and 50.
- Is my answer an odd number? Yes, 25 is an odd number.
- Is my answer greater than 20? Yes, 25 is greater than 20.
- Is my answer less than 40? Yes, 25 is less than 40.
- Is the sum of the digits in my answer equal to 7? Yes, 2 + 5 = 7.

"Now I know that my answer meets all the conditions. I know that Mrs. Truman is 25."

You can extend this example. You can ask students other questions.

Were all the steps necessary? For example, when we constructed the chart of numbers, we could have begun with 3 because we know that her age was not an even number (clue 2). We also knew from clue 5 (the sum of the digits is 7) that the answer would be a two-digit numeral. So, we could have shortened the process even more by beginning the chart with 11.

Now that I know how old Mrs. Truman is, what do I know about her daughter? How old is Mrs. Truman's daughter.

Is there another way to solve this problem?

The Teacher as a Problem Provider

As your students gain confidence and skill, they will need practice. The problems you give students should relate to their reality and involve a variety of situations. Where do you find such problems?

Textbooks. Textbook word problems are often exercises, not problem-solving activities. However, don't discard them completely. You can modify these word problems so that they are problems to be solved.

Take this typical textbook problem:

Rob built 7 model airplanes. He used 3 bottles of paint for each one. How many bottles did he use?

- Add extraneous information to the problem to give students practice in extracting only the information they need to solve the problem.

Rob's model airplane paint set had 5 colors of paint. Each jar contained 2.5 ounces of paint. Rob built 7 model airplanes . . . .

Although students may be familiar with the algorithm used to find a solution, activities of this type give them practice in eliminating irrelevant information, a skill that successful problem solvers must have.

- Omit data to give students practice in identifying what is needed to solve the problem.

Rob built 7 model airplanes. He used the same amount of paint on each airplane. How may jars of paint did he use?

Ask students what information is missing. Ask them what they need to know if they are to find a solution. Ask them how they might obtain the needed information.

- Give students the problem without a question. Let them pose questions. This gives them the opportunity to extend their thinking to the variety of possible problems that can be derived from the same set of data.

- Extend the textbook problem.

Rob spent $8.67 of his allowance for model airplane paint. Some of the paint he bought costs $0.89 a bottle; some costs $1.20. How many bottles of each type did he buy?

Other sources. Your local newspapers are a rich source of ideas for problem solving activities. In every edition of the newspaper, the sports, business, and weather sections contain data for creating problems. You can use newspaper advertisements to create problems dealing with making comparisons and determining the best value.

Ask students to look for situations that require problem-solving skills: installing new carpet, building a dog house, choosing car insurance, determining the probability that the football team will win Friday night's game. Have students write their problems for the class to solve. You might let each student demonstrate the solution to his or her own problem after the class has had a chance to work on it. As students write problems and share in solving their classmate's problems, they see the wide range of problems to which their skills can be applied. They will also begin to detect similarities. "That's like the problem Julie brought in last week, except this time . . . ." Noting the similarity to another problem gives students a starting place; noting the point of divergence gives them the opportunity to begin problem solving.

You can find problems, puzzles and brain teasers in magazines, such as *Games*, or in professional journals, such as *Arithmetic Teacher*, *The Mathematics Teacher* ("Problems of the Month"), and *School Science and Mathematics* ("Problem Department").

Recognize all efforts and approaches. Encourage the students to work together, to discuss the problem, and to share their insights and progress with the class. Students will gain confidence and skill in solving problems — and have fun doing it!